M2 - NAM (HOMEWORK)

COMMUNITY DETECTION

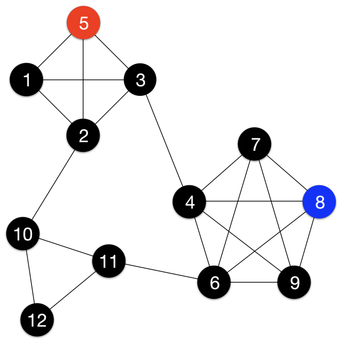

FIGURE 1. Undirected and unweighted graph with 12 nodes.

Exercise 1. Question about random walks

Consider the graph from Figure 1. Imagine that a drunken walker located in the red node aims to go home located in the blue vertex. Since the person is drunk, he walks accross the graph erratically. This is, he chooses the next vertex randomly from its neighbors.

Example: If the walker is at node (11), he will chose one of the three adjacent vertices (10), (6), (12) uniformly at random and transition to the selected node.

The probability to find the walker at any node after he has done t steps is ruled by the following recursive equation:

$$f^{(t)} = Pf^{(t-1)}$$

where

- $f^{(t)}$ is a probability vector of length n for a graph of n nodes. Entry u of this vector, denoted by $f_u^{(t)}$, indicates the probability to find the walker at node u after t steps

- $f^{(0)}$ is the initial condition with entries given as follows:

$$f_u^{(0)} = \begin{cases} 1 & \text{if } u = 5 \\ 0 & \text{otherwise} \end{cases}$$

- P is a matrix in which entry (u, v) is given as follows:

$$P_{uv} = \begin{cases} \frac{1}{d(v)} & \text{if } u \text{ connected } v \\ 0 & \text{if } u \text{ and } v \text{ are disconnected} \end{cases}$$

  where $d(v)$ refers to the degree of node v.

Question 1. The shortest path distance between (5) and (8) is 3. What is the probability that the walker reaches home after 3 steps by walking randomly?

Question 2. Change the recursive equation above and express the probability vector $f^{(t)}$ in terms of $f^{(0)}$ rather than $f^{(t-1)}$

Question 3. Open question: (Use google) How would you show that the probability of finding the walker at any node converges to a value after a long time: $|f_u^{(t)} - f_u^{(t-1)}| \to 0$ as $n \to \infty$

Question 4. Open question: (Use google) How would you calculate the probability of finding the walker at home after an infinite number of steps: $t \to \infty$?